OPTIMAL TRANSFER BETWEEN SLIGHTLY ELLIPTICAL ORBITS WITH AN ADDITIONAL CLAIM OF ACCURACY AND WITH CALCULATION OF NONSPHERICITIY

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16. Abstract

Using the basic system of equations derived in Kovalenko (1972a), the energetically optimal two-pulse transfer between elliptical orbits in the equatorial plane of an axisymmetric planet is examined. Dispersion of flyback is studied as a function of various parameters. A single small parameter is selected, $\alpha(t_u - t_i) = \mu$. The Homann ellipse serves as the simulated transfer.

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OPTIMAL TRANSFER BETWEEN SLIGHTLY ELLIPTICAL ORBITS WITH AN ADDITIONAL CLAIM OF ACCURACY AND WITH CALCULATION OF NONSPHERICITY

A. N. Kovalenko

The basic system of equations employed in this work is the /29
system derived in study [1]. This system describes the energetically optimal two-pulse transfer, with correction for accuracy of implementation, between elliptical orbits in the equatorial plane of an axisymmetric planet.

Study [1] also notes that this system is suitable for deriving solutions in the neighborhood of a known optimal simulated transfer. In this study, the Homann ellipse serves as the simulated transfer, and the selection of a solution from its neighborhood is defined, firstly, by the introduction into the minimized functional (1-1)* of a term characterizing the accuracy of the transfer or the sensitivity to errors in implementing correcting pulses; secondly, by consideration of the nonsphericity for the equatorial plane of an axisymmetric planet; thirdly, by selection in the capacity of limiting slightly elliptic orbits. The methods of solution of problems of this type was demonstrated in the works of V. S. Novoselov [2]-[4].

The analytically derived solution is studied as the sub- /30
ject of the relationship of dispersion of the flyback as a function of various aforementioned factors.

1. Definition of Magnitude of the First Approximation

This study will retain all notations of study [1]. Thus, we consider that e_i, $e_u^\circ \ll 1$. In all, he has a series of small

*The number 1 in the bracket indicates that the particular formula was taken from study [1].

parameters: α, $P_i\sigma^2$, e_i, e_u. The question arises on the selection of a single small parameter. Let us first note, however, that in the system (1-13)-(1-34), the values t_i, t_u only appear as coefficients with α and therefore, for general accuracy of the first power it is sufficient to know t_i, t_u for the zero approximation. And since it is obvious that the zero approximation is the Homann ellipse, the value $(t_u - t_i)$ is easy to calculate for the Homann flyback, and may be considered given. This last fact is important since for the small parameter we will select the value

$$\alpha(t_u - t_i) = \mu,$$

whereof we are not specifying that $(t_u - t_i)$ is calculated for one half turn, and assume several revolutions in the transitional orbit but, naturally, enough to ensure the small value of the quantity μ.

Let us write the solution of system (1-13)-(1-34) in the zero approximation--the Homann ellipse, assuming that $\alpha = 0$, $P_i = 0$, $e_i = e_u = 0$. Without disturbing the generality, let us also assume that $\omega_{i0} = 0$. The top sign in the double transcription relates to the case $p_i < p_u$, the bottom sign--to the case $p_i > p_u$. The first value in curved brackets relates to the case $p_i < p_u$, the second--to $p_i > p_u$:

$$\begin{gathered}
p_0 = \frac{2p_к p_н}{p_к + p_н}, \quad e_0 = \pm \frac{p_к - p_н}{p_к + p_н}, \quad \omega_0 = R_0, \\
f_0^- = E_0^- + \{0, \pi\}, \quad f_0^+ = E_0^+ = \{\pi, 2\pi\}, \quad r_{но} = p_н, \quad r_{ко} = p_к, \\
f_{но} = R_0 + \{0, \pi\}, \quad f_{ко} = R_0 - \omega_{ко} + \{\pi, 2\pi\}, \quad \psi_{10} = \psi_{20} = \pm \frac{\pi}{2}, \\
V_{10} = \pm \frac{\varkappa}{p_н}(p_0^{1/2} - p_н^{1/2}), \quad V_{20} = \mp \frac{\varkappa}{p_к}(p_0^{1/2} - p_к^{1/2}), \\
A_0 = 0, \quad B_0 = \pm \frac{1}{2}, \quad C_0 = 0, \quad D_0 = \pm \frac{1}{2}(1 - e_0^2).
\end{gathered} \tag{1}$$

In the solution of (1), attention is called to the fact that

the value R_0 appearing in f_{i0}, f_{u0} remains undefined. This occurs mathematically as a result of the fact that two conditions of transversality (1-15)-(1-16) degenerate into one; physically, this means that in a round orbit we can arbitrarily select the starting point. It was further shown that this indeterminacy is removed in studying the first approximation.

Considering that our problem is correct in the sense that /31 the small change of boundary conditions leads to a small change in the solution, we write the solution of system (1-13)-(1-34) in the form

$$\begin{gathered} p_0+\mu p',\ e_0+\mu e',\ \omega_0+\mu\omega',\ f_0^-+\mu f^-,\ f_0^++\mu f^+, \\ E_0^-+\mu E^-,\ E_0^++\mu E^+,\ f_{\text{н}0}+\mu f'_{\text{н}},\ f_{\text{к}0}+\mu f'_{\text{к}}, \\ V_{10}+\mu V'_1,\ V_{20}+\mu V'_2,\ \psi_{10}+\mu\psi'_1,\ \psi_{20}+\mu\psi'_2,\ H_0+\mu H',\ T_0+\mu T', \\ A_0+\mu A',\ B_0+\mu B',\ C_0+\mu C',\ D_0+\mu D'. \end{gathered} \tag{2}$$

Also assumed is that $e_i = \mu e'_i$, $e_u = \mu e'_u$ and the following notations are introduced:

$$\theta_1=\frac{t_{\text{н}}-t_0}{t_{\text{к}}-t_{\text{н}}};\quad \theta_2=\frac{t_{\text{к}}-t_0}{t_{\text{к}}-t_{\text{н}}};\quad \theta_3=\frac{\sigma^2}{\pi}. \tag{3}$$

Let us note that the values θ_1, θ_2, and $P_i\theta_3$ have a zero order.

Substituting (2) into (1-13)-(1-34) and using (3), we have a system of equations of the first approximation:
from (1-18)

$$\varkappa e_0\frac{f^{-\prime}}{p_0^{1/2}}-\varkappa e'_{\text{н}}\frac{\sin R_0}{p_{\text{н}}^{1/2}}=-V_{10}\psi'_1, \tag{4}$$

from (1-19)

$$\varkappa e_0 \frac{f^{+\prime}}{p_0^{1/2}} - \varkappa e'_{\text{к}} \frac{\sin(R_0 - \omega_{\text{к}0})}{p_{\text{к}}^{1/2}} = -V_{20}\psi'_2, \tag{5}$$

from (1-20)

$$\varkappa \left[\frac{\pm p'}{2p_{\text{н}} p_0^{1/2}} + (p_0^{1/2} - p_{\text{н}}^{1/2}) \frac{e'_{\text{н}}}{p_{\text{н}}} \cos R_0 \right] = V'_1, \tag{6}$$

from (1-21)

$$\varkappa \left[\frac{\mp p'}{2p_{\text{к}} p_0^{1/2}} + (p_0^{1/2} - p_{\text{к}}^{1/2}) \frac{e'_{\text{к}}}{p_{\text{к}}} \cos(R_0 - \omega_{\text{к}0}) \right] = V'_2, \tag{7}$$

from (1-22)

$$p_{\text{н}} e' = p_0 e'_{\text{н}} \cos R_0 \pm p', \tag{8}$$

from (1-23)

$$p_{\text{к}} e' = p_0 e'_{\text{к}} \cos(R_0 - \omega_{\text{к}0}) \mp p', \tag{9}$$

from (1-24)

$$f^{-\prime} + \omega'_0 + \theta_1 \varkappa \frac{r_{\text{э}}^2}{p^{7/2}} (1 - e_0^2)^{3/2} = f'_{\text{н}} + \theta_1 \varkappa \frac{r_{\text{э}}^2}{p_{\text{н}}^{7/2}} \tag{10}$$

from (1-25)

$$f^{+\prime} + \omega'_0 + \theta_2 \varkappa \frac{r_{\text{э}}^2}{p^{7/2}} (1 - e_0^2)^{3/2} = f'_{\text{к}} + \theta_2 \varkappa \frac{r_{\text{э}}^2}{p_{\text{к}}^{7/2}}, \tag{11}$$

from (1-28)

$$A' + B_0 e_0 f^{-\prime} \pm C' I_0^- = -\psi'_1, \tag{12}$$

from (1-29)

$$-A' - B_0 e_0 f^{+\prime} \pm C' I_0^+ = -\psi'_2, \tag{13}$$

from (1-30)

$$\pm 2B_0e'(1\pm e_0)+B'(1\pm e_0)^2+C'J_0^-(1\pm e_0)+D'=e', \tag{14}$$

from (1-31)

$$\mp 2B_0e'(1\mp e_0)+B'(1\mp e_0)^2+C'J_0^+(1\mp e_0)+D'=-e', \tag{15}$$

from (1-32)

$$E^{-'}=\left(\frac{1\mp e_0}{1\pm e_0}\right)^{1/2}f^{-'}, \tag{16}$$

from (1-33)

$$E^{+'}=\left(\frac{1\pm e_0}{1\mp e_0}\right)^{1/2}f^{+'}, \tag{17}$$

from (1-34)

$$E^{+'}\pm e_0E^{+'}=E^{-'}\mp e_0E^{-'}+\varkappa\frac{r_3^2}{p^{7/2}}(1-e_0^2)^2-$$
$$-\frac{3}{2}\varkappa\frac{p'}{p_0^{5/2}}(1-e_0^2)^{3/2}T_0+3\varkappa\frac{T_0e_0e'}{p_0^{3/2}}(1-e_0^2)^{1/2}+\frac{\varkappa}{p_0^{3/2}}(1-e_0)^{3/2}T', \tag{18}$$

from (1-13)

$$-H'=\left(\sum_{i=1}^{4}P_i\frac{\partial\sigma_i^2}{\partial t_н}+\frac{\partial f^-}{\partial t_н}\sum_{i=1}^{4}P_i\frac{\partial\sigma_i^2}{\partial f^-}+\frac{\partial f^+}{\partial t_н}\sum_{i=1}^{4}P_i\frac{\partial\sigma_i^2}{\partial f^+}\right)', \tag{19}$$

from (1-14)

$$H'=\left(\sum_{i=1}^{4}P_i\frac{\partial\sigma_i^2}{\partial t_к}+\frac{\partial f^-}{\partial t_к}\sum_{i=1}^{4}P_i\frac{\partial\sigma_i^2}{\partial f^-}+\frac{\partial f^+}{\partial t_к}\sum_{i=1}^{4}P_i\frac{\partial\sigma_i^2}{\partial f^+}\right)', \tag{20}$$

from (1-15)

$$-A'\frac{e_0}{p_0^{1/2}}-\frac{1}{\varkappa}\left(\frac{\partial f^-}{\partial f_н}\sum_{i=1}^{4}P_i\frac{\partial\sigma_i^2}{\partial f^-}+\frac{\partial f^+}{\partial f_н}\sum_{i=1}^{4}P_i\frac{\partial\sigma_i^2}{\partial f^+}\right)'-$$
$$-\frac{e'_н}{p_н^{1/2}}\sin R_0\pm\frac{e'_нp_0^{1/2}\left(B_0+\frac{D_0}{1\pm e_0}\right)}{p_н}\sin R_0=0, \tag{21}$$

from (1-16)

$$-A'\frac{e_0}{p_0^{1/2}}+\frac{1}{\varkappa}\left(\frac{\partial f^-}{\partial f_к}\sum_{i=1}^{4}P_i\frac{\partial\sigma_i^2}{\partial f^-}+\frac{\partial f^+}{\partial f_к}\sum_{i=1}^{4}P_i\frac{\partial\sigma_i^2}{\partial f^+}\right)'+$$
$$+\frac{e'_к}{p_к^{1/2}}\sin(R_0-\omega_{к0})\mp\frac{e'_к p_0^{1/2}\sin(R_0-\omega_{к0})}{p_к}\left(B_0+\frac{D_0}{1\mp e_0}\right)=0, \qquad (22)$$

from (1-17)

$$H'=-\varkappa^2 e_0 p_0^{-2}C'. \qquad (23)$$

In formulas (19)-(22), the prime with the round brackets denotes that the particular value is calculated to an accuracy within the order of μ and is divided by μ. In deriving these same formulas, /33 a fact is employed which is that, according to (1-10), (1-11), (1-12)

$$\left(\frac{\partial e}{\partial t_н}\right)_0=\left(\frac{\partial e}{\partial t_к}\right)_0=0,\qquad \left(\frac{\partial p}{\partial t_н}\right)_0=\left(\frac{\partial p}{\partial t_к}\right)_0=0,$$
$$\left(\frac{\partial e}{\partial f_н}\right)_0=\left(\frac{\partial e}{\partial f_н}\right)_0=0,\qquad \left(\frac{\partial p}{\partial f_н}\right)_0=\left(\frac{\partial p}{\partial f_н}\right)_0=0. \qquad (24)$$

The system (4)-(23) is a linear system with respect to e', p', ω_0', $f^{-'}$, $f^{+'}$, f_i', f_u', ψ_1', ψ_2', V_1', V_2', A', B', C', D', H', T', $E^{-'}$, $E^{+'}$ --19 unknowns. And, moreover, in (21), (22) appears no defined value R_0 in the first approximation. Since in the scheduled form equations (19) and (20) are identical, we have a system of 19 equations with 20 unknowns.

Let us proceed to solve this system.

Let us start with the calculation of the value of R_0. In (21), (22), in round brackets with a prime, the order of smallness μ is ensured by the coefficients $P_i\sigma^2$ appearing in these expressions. Thus, all derivatives in (21), (22) and together in (19), (20) can

only be calculated for the zero approximation.

According to (1-11), (1) we find that

$$\left(\frac{\partial f^-}{\partial f_н}\right)_0 = \frac{p_к^2}{p_к^2 - p_н^2} = \pm\frac{(1\pm e_0)^2}{4e_0}, \qquad \left(\frac{\partial f^+}{\partial f_н}\right)_0 = \frac{p_н^2}{p_к^2 - p_н^2} = \pm\frac{(1\mp e_0)^2}{4e_0}. \tag{25}$$

According to (1-12), (1) we yield

$$\left(\frac{\partial f^-}{\partial f_к}\right)_0 = \frac{-p_к^2}{p_к^2 - p_н^2} = \mp\frac{(1\mp e_0)^2}{4e_0}, \quad \left(\frac{\partial f^+}{\partial f_к}\right)_0 = \frac{-p_н^2}{p_к^2 - p_н^2} = \mp\frac{(1\mp e_0)^2}{4e_0}. \tag{26}$$

A comparison of (25), (26) produces the following equation:

$$\left(\frac{\partial f^+}{\partial f_к}\sum_{i=1}^{4} P_i\frac{\partial\sigma_i^2}{\partial f^+} + \frac{\partial f^-}{\partial f_к}\sum_{i=1}^{4} P_i\frac{\partial\sigma_i^2}{\partial f^-}\right)' = -\left(\frac{\partial f^+}{\partial f_н}\sum_{i=1}^{4} P_i\frac{\partial\sigma_i^2}{\partial f^+} + \frac{\partial f^-}{\partial f_н}\sum_{i=1}^{4}\frac{\partial\sigma_i^2}{\partial f^-}\right)'. \tag{27}$$

Correlation (27) makes it possible, after subtracting (22) from (21), to derive a comparatively simple equation for R_0. Solving it, we find that (using (1))

$$\operatorname{ctg} R_0 = \operatorname{ctg}\omega_{к0} - \frac{e'_н}{\sin\omega_{к0}\, e'_к}\cdot\frac{\left[-(1\pm e_0)\left(1\mp\frac{e_0}{2}\right)+(1\pm e_0)^{1/2}\right]}{\left[-(1\pm e_0)\left(1\pm\frac{e_0}{2}\right)+(1\mp e_0)\right]^{1/2}}. \tag{28}$$

Expressions in square brackets in (28) are strictly greater than zero for $0 < e_0 \leq 1$.

Therefore, study of the first approximation permits us to derive not a defined value of R_0 in the zero approximation. /34

Calculation of R_0 according to (28) produces the angle of displacement of the starting point from the direction to the perigee of the initial orbit.

Let us proceed to further solution of system (4)-(23). Formulas (8), (9) yield

$$e' = \frac{p_0 [e'_н \cos R_0 + e'_к \cos (R_0 - \omega_{к0})]}{p_н + p_к}, \tag{29}$$

$$p' = \frac{\mp p_0 [p_к e'_н \cos R_0 - p_н e'_к \cos (R_0 - \omega_{к0})]}{p_н + p_к}. \tag{30}$$

With derivation of p' with respect to (6), (7), V'_1, V'_2 are easily defined. From (4), (12) except for ψ'_1, we find that

$$f^{-\prime} = \frac{\varkappa e'_н \sin R_0 p_н^{-1/2} + V_{10}(A' \pm C' I_0^-)}{e_0 (\varkappa p_0^{-1/2} - B_0 V_{10})}. \tag{31}$$

From (5), (13) except for ψ'_2, we find that

$$f^{+\prime} = \frac{\varkappa e'_к \sin (R_0 - \omega_{к0}) p_к^{-1/2} - V_{20}(A' \mp C' I_0^+)}{e_0 (\varkappa p_0^{-1/2} + B_0 V_{20})}. \tag{32}$$

In (31), (32) appear the still undefined I_0^-, I_0^+, C', A'. The first two values are defined according to (1-4):

$$I_0^- = \mp (1 \pm e_0)^{-2}, \quad I_0^+ = \pm (1 \mp e_0)^{-2}. \tag{33}$$

Here, and in the derivation of formuls (25) and (26), and everywhere hence instead of the values p_i, p_u will be used e_0, p_0 which, in contrast to p_i, p_u are not given, but are expressions written through e_0, p_0 which are simpler in form and easier to study. The shift from e_0, p_0 to p_i, p_u is given in (1). Inverse relations have the form

$$p_н = \frac{p_0}{1 \pm e_0}, \quad p_к = \frac{p_0}{1 \mp e_0}.$$

The values A', C' (together with H') are defined according to (20), (21), (23). At first let us perform auxiliary operations. From (1-10) we derive

$$\left(\frac{\partial f^-}{\partial t_к}\right)_0 = -\left(\frac{\partial f^-}{\partial t_н}\right)_0 = \frac{\varkappa p_0^{1/2}}{p_к^2 - p_н^2} = \pm\frac{\varkappa(1-e_0^2)^2}{4p_0^{3/2}e_0},$$
$$\left(\frac{\partial f^+}{\partial t_к}\right)_0 = -\left(\frac{\partial f^+}{\partial t_н}\right)_0 = \frac{\varkappa p_0^{1/2}}{p_к^2 - p_н^2} = \pm\frac{\varkappa(1-e_0^2)^2}{4p_0^{3/2}e_0}. \qquad (34)$$

Derivatives of f_i, f_u, needed to calculate A' with respect to (21) are already found in (25), (26). The value $(t_u - t_i)_0 = T_0$, not written in (1) as an auxiliary, is defined according to (1-32), (1-33), (1-34):

$$(t_к - t_н)_0 = \frac{\pi p_0^{3/2}}{\varkappa(1-e_0^2)^{3/2}} N, \qquad (35)$$

where N--the number of half turns in the transitional orbit. Ac- /35
cording to Table 5 of study [5] and (1-2), we calculate

$$\left(\frac{\partial\sigma_1^2}{\partial t_к}\right)_0 = -\left(\frac{\partial\sigma_1^2}{\partial t_н}\right)_0 = 18\sigma^2\pi N\frac{\varkappa}{p_0^{3/2}}e_0^2\frac{(1\mp e_0)^{1/2}}{(1\pm e_0)^{3/2}},$$
$$\left(\frac{\partial\sigma_2^2}{\partial t_к}\right)_0 = \left(\frac{\partial\sigma_2^2}{\partial t_н}\right)_0 = 0,$$
$$\left(\frac{\partial\sigma_3^2}{\partial t_к}\right)_0 = \left(\frac{\partial\sigma_3^2}{\partial t_н}\right)_0 = 0, \qquad (36)$$
$$\left(\frac{\partial\sigma_4^2}{\partial t_к}\right)_0 = -\left(\frac{\partial\sigma_4^2}{\partial t_н}\right)_0 = 18\sigma^2\pi N\frac{(1\mp e_0)^{1/2}}{\varkappa p_0^{1/2}(1\pm e_0)^{3/2}},$$
$$\left(\frac{\partial\sigma_1^2}{\partial f^+}\right)_0 = \mp 6\sigma^2\pi N e_0\left[\frac{e_0^2\mp 5e_0+2}{(1\pm e_0)^{5/2}(1\mp e_0)^{3/2}}\right],$$
$$\left(\frac{\partial\sigma_2^2}{\partial f^+}\right)_0 = \pm 6\sigma^2\pi N e_0\left[\frac{3\pm e_0}{(1\pm e_0)^{5/2}(1\mp e_0)^{1/2}}\right],$$
$$\left(\frac{\partial\sigma_3^2}{\partial f^+}\right)_0 = \pm 6\sigma^2\pi N e_0\frac{p_0^3}{\varkappa^2}\left[\frac{4}{(1\pm e_0)^{5/2}(1\mp e_0)^{9/2}}\right],$$
$$\left(\frac{\partial\sigma_4^2}{\partial f^+}\right)_0 = 6\sigma^2\pi N\frac{p_0}{\varkappa^2}\left[\frac{2(2\mp e_0)}{(1\pm e_0)^{5/2}(1\mp e_0)^{3/2}}\right], \qquad (37)$$
$$\left(\frac{\partial\sigma_1^2}{\partial f^-}\right)_0 = \pm 6\sigma^2\pi N e_0\left[\frac{2(1\pm e_0+e_0^2)(1\mp e_0)^{1/2}}{(1\pm e_0)^{9/2}}\right],$$
$$\left(\frac{\partial\sigma_2^2}{\partial f^-}\right)_0 = 0, \quad \left(\frac{\partial\sigma_3^2}{\partial f^-}\right)_0 = 0,$$
$$\left(\frac{\partial\sigma_4^2}{\partial f^-}\right)_0 = -6\sigma^2\pi N\frac{p_0}{\varkappa^2}\left[\frac{2(e_0^2\pm e_0+2)}{(1\pm e_0)^{9/2}(1\mp e_0)^{1/2}}\right].$$

Now, according to (20), we can define H' in the form

$$H' = \frac{3}{2}\cdot\frac{\pi N\varkappa e_0(1\mp e_0)^{1/2}}{(1\pm e_0)^{5/2}p_0^{3/2}}(e_0^3\pm 13e_0^2+19e_0\mp 1)P_1\theta_3+$$
$$+\frac{3}{2}\cdot\frac{\pi N\varkappa(1\mp e_0)^{3/2}(3\pm e_0)}{(1\pm e_0)^{1/2}p_0^{3/2}}P_2\theta_3+\frac{6\pi Np_0^{3/2}}{\varkappa(1\pm e_0)^{1/2}(1\mp e_0)^{5/2}}P_3\theta_3+$$
$$+\frac{6\pi N(1\mp e_0)^{1/2}(5\pm 3e_0)}{\varkappa(1\pm e_0)^{5/2}p_0^{1/2}}P_4\theta_3. \qquad (38)$$

From (21), using (37), 25), we find that

$$A' = \frac{e_n'}{e_0}\sin R_0\left[(1\pm e_0)\left(1\mp\frac{e_0}{2}\right)-(1\pm e_0)^{1/2}\right]-$$
$$-\frac{3}{2}\pi N\frac{p_0^{1/2}(1\mp e_0)^{1/2}(e_0\pm 7)}{\varkappa(1\pm e_0)^{5/2}}P_1\theta_3-\frac{3}{2}\pi N\frac{p_0^{1/2}(1\mp e_0)^{3/2}(3\pm e_0)}{\varkappa e_0(1\pm e_0)^{5/2}}P_2\theta_3-$$
$$-\frac{6\pi Np_0^{7/2}}{\varkappa^3 e_0(1\pm e_0)^{5/2}(1\mp e_0)^{5/2}}P_3\theta_3+\frac{12\pi NP_0^{3/2}}{e_0\varkappa^3(1\mp e_0)^{1/2}(1\pm e_0)^{5/2}}P_4\theta_3, \qquad (39)$$

and C' is defined with respect to H' through (23)

$$C' = -H'\frac{p_0^2}{e_0\varkappa^2}. \qquad (40)$$

Formulas (33), (38)-40) permit us to fully define $f^{-'}$, $f^{+'}$ with respect to (31), (32). /36

The values ψ_1', ψ_2' are conveniently defined according to (12), (13).

As was already indicated, system (4)-(23) is a system of 19 equations with 20 unknowns. The values f_i', f_u' and ω_0' appear only in equations (10), (11) and consequently, can not be defined all at the same time. This circumstance has independent value and is associated with the fact that the Homann flyback, in whose neighborhood the solution is sought is, due to round boundary orbits, a degenerated case of an optimal flyback between elliptical orbits. With this same circumstance was associated the indeterminacy of the value R_0 in the zero approximation.

Therefore, for example, from (10), (11) it is convenient to define the difference

$$f'_{\text{н}}-f'_{\text{к}}=f^{+\prime}-f^{-\prime}+\frac{\varkappa r_3^2}{p_0^{7/2}}(1-e_0^2)^{3/2}-\varkappa r_3^2\left(\frac{\theta_2}{p_{\text{к}}^{7/2}}-\frac{\theta_1}{p_{\text{н}}^{7/2}}\right), \tag{41}$$

without defining f'_u and f'_i individually. Restricted to the first approximation, we can also posit one of these values as equal to zero.

The value ω'_0 is found, for example, according to (10). There is still a problem on defining θ_1, θ_2. According to (3), for this we must know t_i, t_u. These can be found by knowing R_0 from the Kepler equation for slightly elliptical orbits, e.g., in study [8].

The values B', D' can be defined according to (14), (15), but they are auxiliary and their definition is not obligatory. $E^{-\prime}$, $E^{+\prime}$ are found according to (16), (17). From (19) is defined T'

$$T'=\frac{p_0^{3/2}}{\varkappa(1-e_0^2)^{3/2}}\left[\frac{(1\pm e_0)^{3/2}}{(1\mp e_0)^{1/2}}f^{+\prime}-\frac{(1\mp e_0)^{3/2}}{(1\pm e_0)^{1/2}}f^{-\prime}\right]-$$
$$-\frac{r_3^2}{p_0^2}(1-e_0^2)^{1/2}+\frac{3}{2}T_0\left(\frac{p'}{p_0}-\frac{2e_0e'}{1-e_0^2}\right). \tag{42}$$

Therefore, the solution of system (4)-(23) is concluded.

Let us make a quick analysis of the solution derived. In the problem of this study a series of factors is taken into account: the precision of flyback, the nonsphericity of the planet, the slightly elliptical form of the boundary orbits. Let us examine the effect of all these factors on the solution derived with an accuracy of μ. The statement of the problem itself envisions that all these factors have an order by one unit less than the

total energy expenditures. And it is obvious that the zero approximation of (1), aside from the presence therein of the value R_0--the result of the slight ellipticity of boundary orbits--in now way reflects the factors in question. In the first approximation they all appear to different degrees. The values e', p', V_1', V_2' according to (29), (30), (6), (7) are affected only by the slight ellipticity. Formulas (6), (7) show that the problem of enhancing the accuracy of flyback and calculation of nonsphericity does not demand additional power losses of the first order. /37

The values $f^{-'}$, $f^{+'}$ through A'H' (cf. (31), (32)) are already closely associated with the problem of improving accuracy; and through them are associated with accuracy ψ_1', ψ_2', $E^{-'}$, $E^{+'}$ (cf. (12), (13), (16), (17)). And only in the values f_i', f_u', ω_0', and T' does the calculation of nonsphericity have an effect. The latter is of interest to explain physically. It is extremely obvious that nonsphericity affects flyback time. Actually, according to (1-5), secular variation is undergone by mean anomaly M_0, which is closely connected through the Kepler equation with the time of motion. Less apparent is the fact that the value $f_u' - f_i'$ is a function of nonsphericity, but not $f^{+'} - f^{-'}$. The fact of the matter here is that according to (1-5), the values f_i, f_u, f^+, f^- are measured from moving pericenters, but f^+ and f^- are measured from one pericenter; f_u and f_i--from different pericenters; and a different osculating motion of the pericenters of the initial and ultimate orbit is also ensured according to (10), (11) by this property.

2. Studying the Dispersions of the Found Flyback

The derivation and study of dispersions of variance of values v_r, v_ϕ, r, ϕ in a finite orbit for the found solution in terms of a number of causes is useful.

First of all, the theory of solution of variation problems

of this type [6], [7] is based on a study of the equality to zero of the linear portion of the functional increment of type (1-1); and this is only a needed condition of the extremum and presumes the unproven fact that small changes in the boundary conditions of our problem will lead to a small change in the solution. Thus, the verification of a decrease in dispersions in cases where their physical decrease is presumed is indirect proof of the validity of this means of problem-solving.

Secondly, as in section 1, study of dispersions permits us to evaluate the extent and form of effect of diverse factors of the problem on the behavior of the dispersion.

The dispersions of σ_i^2 are calculated according to (1-2), Table 5 of study [5] and formulas in section 1. Two points of view are possible on the order of dispersion of σ_i^2: 1) although $P_i\sigma^2$ are small, but σ^2 is not small and then the zero order of magnitudes of σ_i^2 characterizes flyback along the Homann ellipse, and the first order corresponds to the more specific solution of section 1; 2) $P_i\sigma^2$ and σ^2 are small. Then, the first order of σ_i^2 corresponds to the Homann flyback, and the second--to the more precise solution of section 1. In both cases, the expressions in square brackets in (1-2) are calculated with an accuracy of μ, and the differences between the first and second points of view will henceforth not be made.

Let us also indicate the additivity of the entry of factors of nonsphericity, accuracy and slight ellipticity in the dispersion of σ_i^2, which is based on the fact that all these factors have an identical first order of smallness and notably with such order occurs the calculation of square brackets in (1-2). /38

After transformations we find that

$$\sigma_1^2=\sigma^2\left[1+\frac{(1\mp e_0)^4}{(1\pm e_0)^4}+\frac{9\pi^2N^2e_0^2}{(1\pm e_0)^3(1\mp e_0)}\pm\right.$$
$$\pm\frac{\mu 6\pi Ne_0(2\pm e_0)(1\mp e_0)^{3/2}}{(1\pm e_0)^{9/2}}f^{-\prime}\mp\frac{\mu 12\pi Ne_0(1\mp 4e_0-e_0^2)}{(1\mp e_0)^{3/2}(1\pm e_0)^{5/2}}f^{+\prime}+$$
$$\left.+\frac{\mu 12\varkappa r_3^2\pi Ne_0^2(1\mp e_0)(2\mp e_0)}{p_0^{7/2}(1\pm e_0)^2}\mp\frac{\mu 18e'\pi^2N^2(1\mp e_0+e^2)}{(1\mp e_0)^2(1\pm e_0)^3}\right], \tag{43}$$

$$\sigma_2^2=\sigma^2\left[1+\frac{(3\pm e_0)^2}{(1\pm e_0)^2}\pm\right.$$
$$\left.\pm\frac{\mu 6\pi Ne_0(3\pm e_0)}{(1\pm e_0)^{5/2}(1\mp e_0)^{1/2}}f^{+\prime}\mp\frac{\mu 4(3\pm e_0)e'}{(1\pm e_0)^3}\right], \tag{44}$$

$$\sigma_3^2=\sigma^2\left[\frac{16p_0^3}{\varkappa^2(1\mp e_0)^4(1\pm e_0)^2}\pm\frac{\mu 24p_0^3\pi Ne_0}{\varkappa^2(1\mp e_0)^{9/2}(1\pm e_0)^{5/2}}f^{+\prime}+\right.$$
$$\left.+\frac{\mu 48p_0^3p'}{\varkappa^2(1\mp e_0)^6(1\pm e_0)^2}\pm\frac{\mu 32p_0^3(1\pm 3e_0)e'}{\varkappa^2(1\mp l_0)^5(1\pm e_0)^3}\right], \tag{45}$$

$$\sigma_4^2=\sigma^2\left[\frac{16p_0}{\varkappa^2(1\pm e_0)^4}+\frac{9p_0\pi^2N^2}{\varkappa^2(1\pm e_0)^3(1\mp e_0)}+\right.$$
$$+\frac{\mu 6p_0\pi N(7\pm e_0)}{\varkappa^2(1\pm e_0)^{5/2}(1\mp e_0)^{3/2}}f^{+\prime}-\frac{\mu 6\pi Np_0(7\pm 2e_0-e_0^2)}{\varkappa^2(1\mp e_0)^{1/2}(1\pm e_0)^{9/2}}f^{-\prime}+$$
$$+\frac{\mu 6\pi Nr_3^2p_0^{3/2}(7\mp 3e_0-2e_0^2)}{\varkappa(1\pm e_0)^3(1\mp e_0)}+\frac{\mu 9\pi^2N^2(1\mp e_0)^2p'}{\varkappa^2(1-e_0^2)^3}\mp$$
$$\left.\mp\frac{\mu 18\pi^2N^2(1\pm 3e_0)p_0e'}{\varkappa^2(1\mp e_0)^2(1\pm e_0)^4}\right]. \tag{46}$$

In (43)-(46), the terms without μ correspond to flyback along the Homann ellipse. The terms with r_e^2, present only in (43) and (46), are derived from the calculation of nonsphericity. It is easy to see that they are strictly greater than zero for $0 \leq e_0 < 1$ and hence follows the conclusion that calculation of nonsphericity in the particular problem increases the dispersions of variance in terms of the transversal constituent velocity and the value of the radius-vector.

Let us now discard the condition of nonsphericity and slight ellipticity of the boundary orbits:

$$a = 0,\ e_i = e_u = 0.$$

This happens to be the case when he have a problem of specifying flyback along the Homann ellipse. If the methods of solution of the problem are true, then we must derive a reduction in dispersion.

From (21), (22) we find that

$$f^{-\prime}=\frac{V_{10}(A'\pm C'I_0^-)}{e_0(\varkappa p_0^{-1/2}-B_0V_{10})},\quad f^{+\prime}=\frac{-V_{20}(A'\mp C'I_0^+)}{e_0(\varkappa p_0^{-1/2}-B_0V_{20})}. \tag{47}$$

According to (1) it is easy to see that V_{10}, $V_{20} > 0$. The denominators in (47) are also greater than 0, which can be shown by the following transformations /39

$$\frac{\varkappa}{p_0^{1/2}}-B_0V_{10}=\frac{\varkappa}{p_0^{1/2}}-\frac{\varkappa}{2p_н}(p_0^{1/2}-p_н^{1/2})=\frac{\varkappa}{2p_0^{1/2}}[1\mp e_0+(1\pm e)^{1/2}]>0,$$
$$\frac{\varkappa}{p_0^{1/2}}+B_0V_{20}=\frac{\varkappa}{p_0^{1/2}}-\frac{\varkappa}{2p_к}(p_0^{1/2}-p_к^{1/2})=\frac{\varkappa}{2p_0^{1/2}}[1\pm e_0+(1\mp e)^{1/2}]>0.$$

Therefore, analysis of the sign of $f^{-\prime}$, $f^{+\prime}$ reduces to the study of expressions

$$(A'\pm C'I_0^-),\quad -(A'\mp C'I_0^+).$$

According to (33), (40) they are transformed to the form

$$A'+\frac{H'p_0^2}{e_0\varkappa^2(1\pm e_0)^2},\qquad -\left(A'+\frac{H'p_0^2}{e_0\varkappa^2(1\mp e_0)^2}\right). \tag{48}$$

Now let us posit $P_2 = P_3 = P_4 = 0$. This is simple in meaning--only errors with respect to the radial constituent velocity are taken into account.

According to (38), (39), we find for this case the values of (48)

$$A' + \frac{H' p_0^2}{e_0 \varkappa^2 (1 \pm e_0)^2} = \mp 6\pi N \frac{p_0^{1/2}}{\varkappa} \frac{(2 \pm e_0)(1 \mp e_0)^{3/2}}{(1 \pm e_0)^{9/2}} P_1 \theta_3,$$

$$-\left(A' + \frac{H' p_0^2}{e_0 \varkappa^2 (1 \mp e_0)^2}\right) = \pm 12\pi N \frac{p_0^{1/2}}{\varkappa} \frac{1 \mp 4e_0 - e_0^2}{(1 \pm e_0)^{5/2} (1 \mp e_0)^{3/2}} P_1 \theta_3. \quad (49)$$

Before substituting (47), (49) in (43), let us note that $e_i = e_u = 0$ according to (29), (30) entails $p' = e' = 0$; hence in (43) the last term vanishes; the penultimate term vanishes for a central field.

In summary we have

$$\sigma_1^2 = \sigma^2 \left[1 + \frac{(1 \mp e_0)^4}{(1 \pm e_0)^4} + \frac{9\pi^2 N^2 e_0^2}{(1 \pm e_0)^3 (1 \mp e_0)} - \frac{36\pi^2 N^2 p_0^{1/2} (2 \pm e_0)^2 (1 \mp e_0)^3 V_{10} P_1 \sigma^2}{\varkappa (1 \pm e_0)^9 (\varkappa p_0^{-1/2} - B_0 V_{10})} - \frac{144\pi^2 N^2 p_0^{1/2} (1 \mp 4e_0 - e_0^2)^2 V_{20} P_1 \sigma^2}{\varkappa (1 \pm e_0)^5 (1 \mp e_0)^3 (\varkappa p_0^{-1/2} + B_0 V_{20})}\right]. \quad (50)$$

By analogy, let us examine the case $P_1 = P_3 = P_4 = 0$ (errors with respect to v_ϕ). According to (38), (39) we find that

$$-\left(A' + \frac{H' p_0^2}{\varkappa^2 e_0 (1 \mp e_0)^2}\right) = \mp \frac{6\pi N p_0^{1/2} (3 \pm e_0) P_2 \theta_3}{\varkappa (1 \pm e_0)^{5/2} (1 \mp e_0)^{1/2}} \quad (51)$$

and with accuracy to μ

$$\sigma_2^2 = \sigma^2 \left[1 + \frac{(3 \pm e)^2}{(1 \pm e_0)^2} - \frac{36\pi^2 N^2 p_0^{1/2} (3 \pm e_0)^2 V_{20} P_2 \sigma^2}{\varkappa (1 \pm e_0)^5 (1 \mp e_0) (\varkappa p_0^{-1/2} + B_0 V_{20})}\right]. \quad (52)$$

The case $P_1 = P_2 = P_4 = 0$ (error with respect to the radius-vector). /40
According to (38), (39) we find that

$$-\left(A' + \frac{H' p_0^2}{\varkappa^2 e_0^2 (1 \mp e_0)^2}\right) = \mp \frac{24\pi p_0^{7/2} P_3 \theta_3}{\varkappa^3 (1 \pm e_0)^{5/2} (1 \mp e_0)^{5/2}}. \quad (53)$$

Substituting (47), (53) into (45), we find that

$$\sigma_3^2=\sigma^2\left[\frac{16p_0^3}{x^2(1\mp e_0)^4(1\pm e_0)^2}-\frac{576\pi^2N^2p_0^{13/2}V_{20}P_3\sigma^2}{x^5(1\pm e_0)^5(1\mp e_0)^9(xp_0^{-1/2}+B_0V_{20})}\right]. \tag{54}$$

The case $P_1 = P_2 = P_3 = 0$ (errors with respect to the angle ϕ).

According to (38), (39), we find that

$$A'+\frac{H'p_0^2}{x^2e_0(1\pm e_0)^2}=\frac{6\pi Np_0^{5/2}(7\pm 2e_0-e_0^2)}{e_0x^3(1\pm e_0)^{9/2}(1\mp e_0)^{1/2}}P_4\theta_3,$$
$$-\left(A'+\frac{H'p_0^2}{x^2e_0(1\mp e_0)^2}\right)=-\frac{6\pi Np_0^{3/2}(7\pm e_0)P_4\theta_3}{e_0x^3(1\mp e_0)^{3/2}(1\pm e_0)^{5/2}}. \tag{55}$$

Substituting (47), (54) into (46) we find that

$$\sigma_4^2=\sigma^2\left[\frac{16p_0}{x^2(1\pm e_0)^4}+\frac{9\pi^2N^2p_0(1\mp e_0)^{3/2}}{x^3(1\pm e_0)^{1/2}}-\right.$$
$$-\frac{36\pi^2N^2p_0^{5/2}(7\pm e_0)^2V_{20}P_4\sigma^2}{x^5e_0^2(1\mp e_0)^3(1\pm e_0)^5(xp_0^{-1/2}+B_0V_{20})}-\left.\frac{36\pi^2N^2p_0^{5/2}(7\pm 2e_0-e_0^2)^2V_{10}P_4\sigma^2}{x^5e_0^2(1\mp e_0)(1\pm e_0)^9(xp_0^{-1/2}-B_0V_{10})}\right] \tag{56}$$

Formulas (50), (52), (54), (56) show a reduction in dispersions of variance of the values v_r, v_ϕ, or, ϕ in a finite orbit.

In the general case $P_1 \neq 0$, $P_2 \neq 0$, $P_3 \neq 0$, $P_4 \neq 0$, we study the reduction of expression $\sum_{i=1}^{4} P_i\sigma_i^2$.

Let us write, according to (43)-(46), the expressions with f^{-}, f^{+} appearing in $\sum_{i=1}^{4} P_i\sigma_i^2$:

$$\mu f^{-}\left[\pm\frac{6\pi Ne_0(2\pm e_0)(1\mp e_0)^{3/2}}{(1\pm e_0)^{9/2}}P_1-\frac{6\pi Np_0(7\pm 2e_0-e_0^2)P_4}{x^3(1\mp e_0)^{1/2}(1\pm e_0)^{9/2}}\right], \tag{57}$$

$$\mu f^{+'}\left[\mp\frac{12\pi Ne_0(1\mp 4e_0-e_0^2)}{(1\mp e_0)^{3/2}(1\pm e_0)^{5/2}}P_1\pm\frac{6\pi Ne_0(3\pm e_0)P_2}{(1\pm e_0)^{5/2}(1\mp e_0)^{1/2}}\pm\right.$$
$$\left.\pm\frac{24p_0^3\pi Ne_0P_3}{\varkappa^2(1\mp e_0)^{9/2}(1\pm e_0)^{5/2}}+\frac{6\pi Np_0(7\pm e_0)P_4}{\varkappa^2(1\pm e_0)^{5/2}(1\mp e_0)^{3/2}}\right] \qquad (58)$$

Now, according to (38), (39), and (47), or using already found by parts (49), (51), (53), (55), let us write out the values $f^{-'}$ and $f^{+'}$ (remember that this is still being under the assumption that $a = 0$, $e_i = e_u = 0$):

$$f^{-'}=\frac{V_{10}}{e_0(\varkappa p_0^{-1/2}-B_0V_{10})}\left[\mp\frac{6\pi Np_0^{1/2}(2\pm e_0)(1\pm e_0)^{3/2}P_1\theta_3}{\varkappa(1\pm e_0)^{9/2}}+\right.$$
$$\left.+\frac{6\pi Np_0^{3/2}(7\pm 2e_0-e_0^2)P_4\theta_3}{\varkappa^3e_0(1\pm e_0)^{9/2}(1\mp e_0)^{1/2}}\right], \qquad (59)$$

$$f^{+'}=\frac{V_{20}}{e_0(\varkappa p_0^{-1/2}+B_0V_{20})}\left[\pm\frac{12\pi Np_0^{1/2}(1\mp 4e_0-e_0^2)P_1\theta_3}{\varkappa(1\pm e_0)^{5/2}(1\mp e_0)^{3/2}}\mp\right.$$
$$\mp\frac{6\pi Np_0^{1/2}(3\pm e_0)P_2\theta_3}{\varkappa(1\pm e_0)^{5/2}(1\mp e_0)^{1/2}}\mp\frac{24\pi Np_0^{7/2}P_3\theta_3}{\varkappa^3(1\mp e_0)^{9/2}(1\pm e_0)^{5/2}}-$$
$$\left.-\frac{6\pi Np_0^{3/2}(7\pm e_0)P_4\theta_3}{\varkappa^3e_0(1\pm e_0)^{5/2}(1\mp e_0)^{3/2}}\right]. \qquad (60)$$ /41

Comparison of (59), (60) with (57), (58) shows that the value $\sum_{i=1}^{4}P_i\sigma_i^2$ also decreases with respect to the Homann ellipse. Our attention is drawn to the fact that the specification of full squares in this expression (comparison of (57) with (59), (58) with (60)) exists. This property was noted in study [3] and requires deeper study. Generally, within wide ranges can we compare the results of study [3] and this one, since the particular case of this particular study ($a = 0$, $e_i = e_u = 0$) coincides with a certain particular case of study [3]. All possible comparisons were made and the results, despite some distinction in methods, coincided. Let us note in conclusion that the slightly elliptical

orbit in a complex manner affects the behavior of dispersions and can both reduce and increase them.

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